**FB1:**

a) Use (i) the Midpoint Rule and (ii) Simpson’s Rule to approximate the integral

using n = 10 grid intervals. Round your answer to 6 decimal places.

b) Compute the integral exactly using antiderivatives and decide which method from part (a) is closest to the true value.

a)

i) By dividing the interval [2, 4] into 10 grid intervals, the width of each interval would be

The intervals would then be [2, 2.2], [2.2, 2.4], …, [3.8, 4]. The Midpoint Rule states that

for some positive integer *i*, where . Thus, the list of midpoints would be Therefore, approximation using the Midpoint Rule would be

ii) Simpson’s Rule states that

where are the values of the function (in this case ) for each in 2, 2.2, …, 4.

When substituting for 0.2 as found in part (a)(i) and values calculated by inserting each in the above expression, the approximation by Simpson’s Rule becomes

b) Since the antiderivative of is

the definite integral of the function in the interval (2, 4) is

Calculating the absolute error made by the Midpoint Rule |

while the absolute error made by Simpson’s Rule is

Since the result obtained by Simpson’s Rule is the closest to the true value.

**FB2:**

Let be a set, and let and be subsets of . Is it true that

If so provide a proof, and if not provide a counterexample.

Suppose an arbitrary element Then *x* is at least either in A or in Bi for all *i* (or both). Hence, for all *i*, meaning that

Going the other way, suppose an arbitrary element . Then for all *i* (in other words, if for some integer *k* in will not be in the intersection of all instances of. Hence, or for all *i*. Thus,

Since all elements from each side of the equation are in the other, the statement

is proven true.

**FB3:**

Consider the relation on ℝ defined as follows for all ℝ .

Show that is an equivalence relation and describe geometrically how this relation partitions the plane.

To check for **reflexivity**, let an arbitrary ℝ2. Since

the relation is reflexive because

To check for **symmetry**, the condition of the relation can be multiplied by to get

which is the condition for the relation of ; thus, the relation is symmetric.

To check for **transitivity**, let an arbitrary ℝ2. If and assuming

Rearranging both equations, one gets

By comparing both equations, one can see that

which, rearranged, is

the condition for the relation ; thus, the relation is transitive.

When the condition for the relation is rearranged to make a function , the function is

This arrangement shows that the relation partitions the plane into linear equations with gradient 1 and y-axis intersection of